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An Easy Way to Determine the Monthly Mortgage Payment for a Given Mortgage

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As we all know, it is very popular for people to plan on purchasing a residence, especially if they do not currently own one. Ideally, of course, one would hope to purchase the best home one could afford. However, as a first step in the process of determining whether a given piece of real estate may be affordable, one must know what the monthly mortgage payment for that property would be. Unfortunately, the calculation of that dollar amount is not an easy matter for most people, even with the aid of a hand held calculator. The required monthly payment **A** that would pay off a given mortgage amount **P**, after successfully making **n** consecutive monthly payments at an interest rate of **i** per month, is given by the following formula:

$$A = P\,[\ i \,/\, \{ 1 – (1{+}I)^{-n} \}\ ]$$

We define the expression $[\ i \,/\, \{ 1 – (1{+}I)^{-n} \}\ ]$ as the **CRUCIAL FACTOR**.

Happily, it may come as a surprise that the relationship between the monthly interest rate **I** and the **CRUCIAL FACTOR**, for any given time span of a mortgage, may be closely approximated by a linear model. We employed linear regression via a computer statistical package, in order to find the most efficient model for each of four typical mortgage time spans - 15, 20, 25 and 30 years.

More specifically, we chose each of 14 nominal (annual) interest rates in increments of 0.5%, between 3% and 9.5% inclusive (the range of typical annual mortgage rates for at least the past 20 years), and, for each rate, we computed

{10^7 MULTIPLIED BY the CRUCIAL FACTOR} for each mortgage time span. We chose to use **nominal** interest rates and **multiplication** of the CRUCIAL FACTOR by **10 million** for the linear regression analyses, in order to minimize the use of decimals in our linear models to the extent possible, so as to achieve relative ease of the evaluation of our models. In each model, the independent variable is the nominal interest rate **12i** and the dependent variable is the quantity {**10^7MULTIPLIED BY the CRUCIAL FACTOR**}.

The results of these four linear reqression analyses, along with their analysis of variance related statistics, are noted in the accompanying supporting information, and follow below:

(1) FOR **FIFTEEN** YEAR MORTGAGES : **Y = 5443.538 X + 52,104.242**

R^2= 0.999; the t statistic = 111.572; **the P-value = 1.8 (10^{-19})**

(2) FOR **TWENTY** YEAR MORTGAGES : **Y = 5814.492 X + 37,231.352**

R^2= 0.999; the t statistic = 95.534; **the P-value = 1.16 (10^{-18})**

(3) FOR **TWENTY –FIVE** YEAR MORTGAGES : **Y = 6156.963 X + 28,036.055**

R^2= 0.998; the t statistic = 88.05; **the P-value = 3.08 (10^{-18})**

(4) FOR **THIRTY** YEAR MORTGAGES: **Y= 6466.677 X + 21752.198**

R^2 = 0.998; the t statistic = 85.103; **the P-value = 4.62 (10^{-18})**

These four linear models are applicable for ANY mortgage amount, and for any nominal interest rate between 3% and 9.5%.

The extremely high values of $\mathbf{R^2}$ for each of the above linear models and their correspondingly extremely low **P-**values strongly suggest that the use of each of these linear models will produce an excellent approximation to the true monthly mortgage payment. We illustrate the use of each model below:

(1) Suppose a buyer has purchased a property, by establishing a 15 year/ $100,000 mortgage, at a nominal interest rate of 6% compounded monthly. Thus, n = 180 and i = 0.005. Via the use of the CRUCIAL FACTOR, the ACTUAL monthly payment is $843.86. However, we may use model (1) and compute:

$$5443.538(6) + 52,104.242 = 84766.$$

Thus, the approximate monthly mortgage payment is computed as

$$84766(\$100,000)\ (10^{-7}) = \$847.66,$$

which differs from the actual monthly mortgage payment by $3.80. The percentage error = **-** 0.45%.

(2) Suppose a buyer has purchased a property, by establishing a 20 year/ $100,000 mortgage at a nominal interest rate of 3.5% compounded monthly. Thus, n = 240 and i = 0.0029167. Via the use of the CRUCIAL FACTOR, the ACTUAL monthly payment is $579.97. However, we may use model (2) and compute:

$$5814.492(3.5) + 37,231.352 = 57582.$$

Thus, the approximate monthly mortgage payment is computed as

$$57582(\$100,000)\ (10^{-7}) = \$575.82,$$

which differs from the actual monthly mortgage payment by $4.15. The percentage error = 0.72%.

(3) Suppose a buyer has purchased a property, by establishing a 25 year/ $100,000 mortgage at a nominal interest rate of 4% compounded monthly. Thus, n = 300 and i = 0.0033333. Via the use of the CRUCIAL FACTOR, the ACTUAL monthly payment is $527.83. However, we may use model (3) and compute:

$$6156.963(4) + 28036.055 = 52664.$$

Thus, the approximate monthly mortgage payment is computed as

$$52664(\$100,000)\ (10^{-7}) = \$526.64,$$

which differs from the actual monthly mortgage payment by $1.19. The percentage error = 0.23%.

(4) Suppose a buyer has purchased a property, by establishing a 30 year/ $100,000 mortgage, at a nominal interest rate of 5% compounded monthly. Thus, n = 360 and i = 0.0041667. Via the use of the CRUCIAL FACTOR, the ACTUAL monthly payment is $536.83. However, we may use model (4) and compute

$$6466.677(5) + 21752.198 = 54086.$$

Thus, the approximate monthly mortgage payment is computed as

$$54086(\$100,000)\ (10^{-7}) = \$540.86,$$

which differs from the actual monthly mortgage payment by $4.03. The percentage error = -0.75%.

In order to better gauge the relative accuracy of our four linear models, each of these models were used with each of 8 interest rates, varying from 3.5% to 9.5 %. Then, each of those 32 linear approximations were compared with the corresponding actual value of {10^7MULTIPLIED BY the CRUCIAL FACTOR}. The percentage errors of approximation follow below:

PERCENTAGE ERROR COMPARING {10^7TIMES CRUCIAL FACTOR} TO LINEAR APPROXIMATION

		MORTGAGE TIME SPAN			
		15 YEARS	20 YEARS	25 YEARS	30 YEARS
	3.5%	0.46	0.72	0.93	1.16
	4.0%	0.12	0.18	0.23	0.26
NOMINAL	5.0%	-0.31	-0.47	-0.62	-0.75
	6.0%	-0.45	-0.66	-0.85	-1.00
INTEREST	7.5%	-0.25	-0.35	-0.42	-0.47
	8.0%	-0.09	-0.12	-0.14	-0.15
RATE	9.0%	0.33	0.46	0.56	0.63
	9.5%	0.58	0.80	0.96	1.07
AVERAGE		0.32%	0.47 %	0.59%	0.69%

ABSOLUTE ERROR

Finally, the **grand** average absolute error is only **0.52%**.
Thus, it is apparent that these four linear models provide a relatively simple and highly accurate method of determining the required monthly payment for a mortgage.

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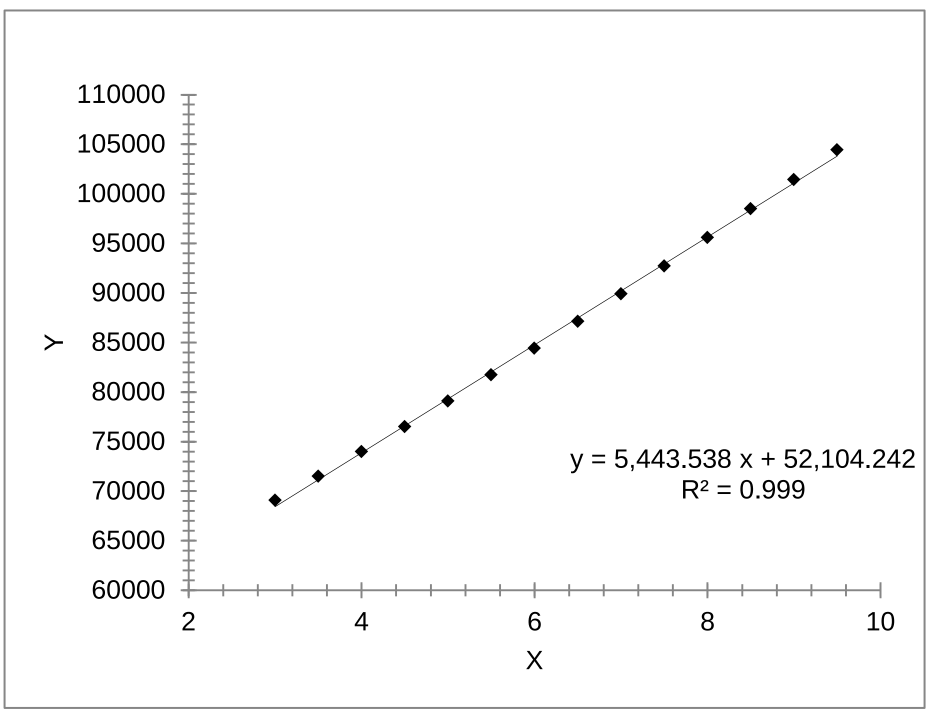


Regression Analysis

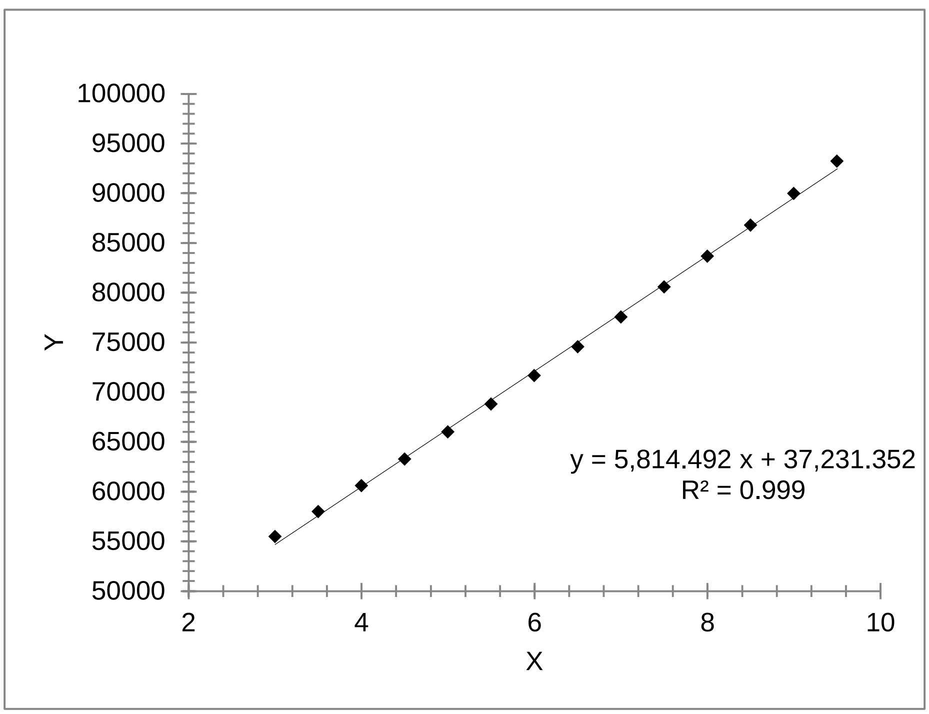
r^2	0.999	n	14
r	1.000	k	1
Std. Error	367.947	Dep. Var.	**Y**

ANOVA table

Source	*SS*	*df*	*MS*	*F*	*p-value*
Regression	1,685,326,312.1154	1	1,685,326,312.1154	12448.37	1.80E-19
Residual	1,624,623.0989	12	135,385.2582		
Total	1,686,950,935.2143	13			

Regression output

					confidence interval	
variables	*coefficients*	*std. error*	*t (df=12)*	*p-value*	*95% lower*	*95% upper*
Intercept	52,104.2418					
X1	5,443.5385	48.7893	111.572	1.80E-19	5,337.2356	5,549.8413



Regression Analysis

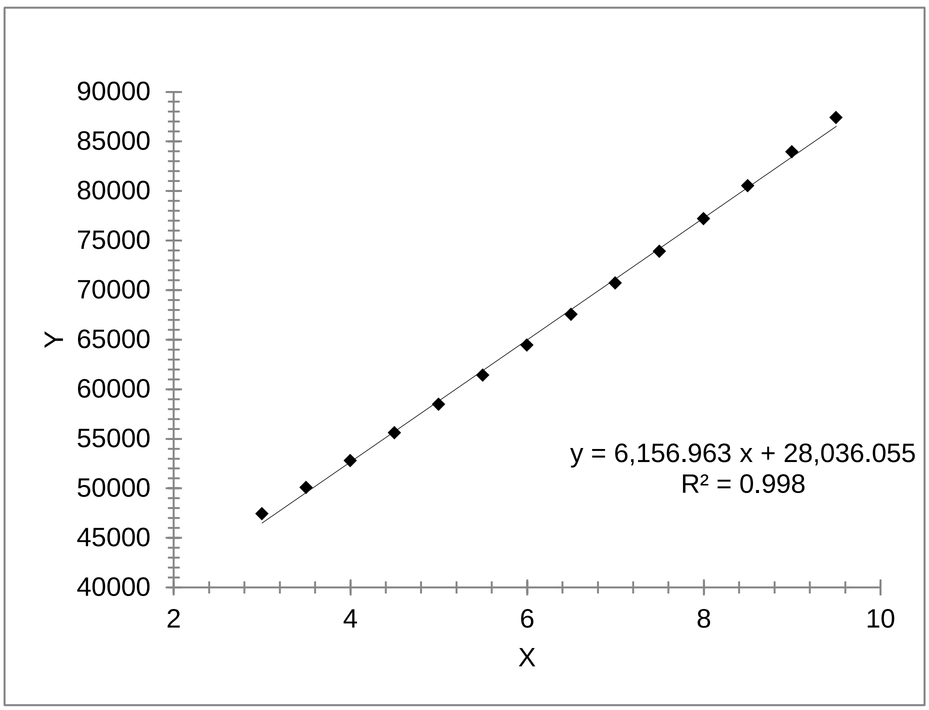
r^2	0.999	n	14
r	0.999	k	1
Std. Error	459.004	Dep. Var.	**Y**

ANOVA table

Source	*SS*	*df*	*MS*	*F*	*p-value*
Regression	1,922,848,245.2846	1	1,922,848,245.2846	9126.66	1.16E-18
Residual	2,528,217.6440	12	210,684.8037		
Total	1,925,376,462.9286	13			

Regression output

					confidence interval	
variables	*coefficients*	*std. error*	*t (df=12)*	*p-value*	*95% lower*	*95% upper*
Intercept	37,231.3516					
X1	5,814.4923	60.8634	95.534	1.16E-18	5,681.8824	5,947.1022



Regression Analysis

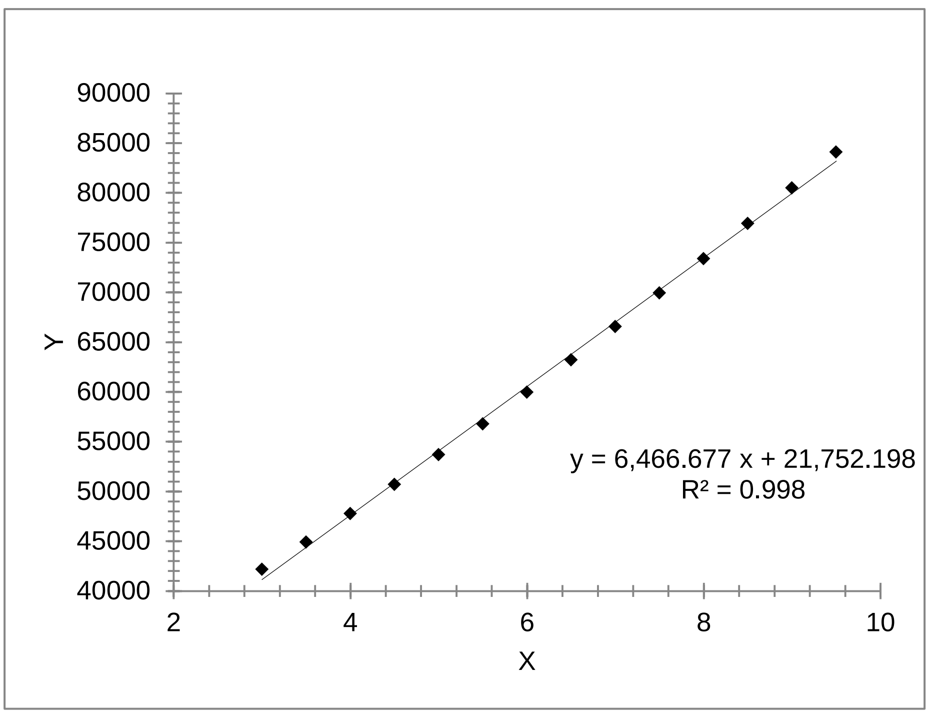
r^2	0.998	n	14
r	0.999	k	1
Std. Error	527.347	Dep. Var.	**Y**

ANOVA table

Source	*SS*	*df*	*MS*	*F*	*p-value*
Regression	2,156,028,244.7044	1	2,156,028,244.7044	7752.84	3.08E-18
Residual	3,337,142.2242	12	278,095.1853		
Total	2,159,365,386.9286	13			

Regression output

					confidence interval	
variables	*coefficients*	*std. error*	*t (df=12)*	*p-value*	*95% lower*	*95% upper*
Intercept	28,036.0549					
X1	6,156.9626	69.9256	88.050	3.08E-18	6,004.6079	6,309.3174



Regression Analysis

r^2	0.998	n	14
r	0.999	k	1
Std. Error	573.057	Dep. Var.	**Y**

ANOVA table

Source	*SS*	*df*	*MS*	*F*	*p-value*
Regression	2,378,393,655.5615	1	2,378,393,655.5615	7242.49	4.62E-18
Residual	3,940,733.3670	12	328,394.4473		
Total	2,382,334,388.9286	13			

Regression output

					confidence interval	
variables	*coefficients*	*std. error*	*t (df=12)*	*p-value*	*95% lower*	*95% upper*
Intercept	21,752.1978					
X1	6,466.6769	75.9866	85.103	4.62E-18	6,301.1163	6,632.2376