A Tutorial on Arbitrage

E. Tylor Claggett, Ph.D., CFA
Professor of Finance
Perdue School of Business
Salisbury University
Salisbury, Maryland 21801, USA

I. Introduction

According to Investo*pedia*, "Arbitrage is the simultaneous purchase and sale of an asset to profit from a difference in the price. It is a trade that profits by exploiting the price differences of identical or similar financial instruments on different markets or in different forms. Arbitrage exists as a result of market inefficiencies."[1]

Often senior business students, as well as seasoned investment professionals, do not have a clear idea of why arbitrage opportunities occur or how to take advantage of them when they are observed. Furthermore, the term "arbitrage" is often misused or used incorrectly. When most people use the word arbitrage, they do not mean "perfect" arbitrage. There is probably no such thing as a perfect arbitrage opportunity in the real-world. Nevertheless, it is often beneficial to recall the definition of perfect arbitrage which will be done in a later section. This paper is intended to discuss arbitrage (perfect and real-world), outline the ways an investor (arbitrager) can recognize and then take advantage of arbitrage opportunities, and show the reader several examples of plausible real-world situations that have arbitrage potential.

II. Arbitrage Aspects

Arbitrage opportunities can be created in many ways. But, regardless of circumstances, behind every arbitrage opportunity, there is a 'market imperfection.' Said differently, in perfect markets, there are no arbitrage opportunities.[2] There are several important assumptions associated with perfect markets that, if not present, can create an arbitrage opportunity. Some of the major perfect market assumed characteristics are: asset prices are consistent according to each's risk versus expected return profile; all market participants have total and complete information; there are no barriers to capital movement across countries, among asset classes, etc.; no one market participant can change asset prices by his or her buying and/or selling decisions; no transactions costs; and no economic 'surpluses' i.e. extra ordinary profits. Violations of these assumptions cause assets to be "mispriced' and mispriced assets lead to arbitrage opportunities.

A simple example is when a share of common stock is selling for one price in a given market and the same share of stock can be sold for a slightly different price in another market. In such a case, if it is possible, an arbitrager would buy the stock in the market with the lower price and sell it instantaneously in the market with the higher price. The price difference would be realized as an arbitrage 'profit.'

However, there could be situations that are much more complex and that involve multiple assets that also present an arbitrager with opportunities to capture arbitrage profits. An example would be the existence at a point in time of three (or possibly more) currency exchange rates that are not in equilibrium. Not in equilibrium means an investor could profit instantaneously or nearly instantaneously by executing a series of currency exchanges and end up with more cash than when he or she started. Another example could be two debt instruments of the same risk class, but with slightly different maturities and significantly different yields until maturity. The two yields until maturity would be current 'spot' yields. These two yields would imply the existence of an equilibrium future spot yield.

[1] http://www.investopedia.com/terms/a/arbitrage.asp
[2] https://en.wikipedia.org/wiki/Perfect_competition

The idea is an investor would be indifferent between investing in the longer maturing debt instrument and investing in the shorter debt instrument plus rolling said proceeds into a similar risk debt instrument yielding the equilibrium future spot rate for the time period between the longer and shorter maturities of the original debt instruments. If these examples seem confusing, later in this paper, specific examples with numbers will follow.

III. Perfect Arbitrage

A perfect arbitrage opportunity has three features: 1) none of the investor's money in the deal, 2) no risk (of any type) in the deal, and 3) a positive return in the deal. It is almost impossible to imagine such a scenario in the real-world. That being said, if perfect arbitrage opportunities were available and recognized by investors, one investor could be as "large" as they cared to be. Thus, just their solo buying and selling activities could move related asset prices. These price changes would guide markets back toward equilibrium.

Typically, a real-world arbitrage opportunity features less than the normal initial investment or less than a corresponding level of risk or more than the normal amount of expected return. So real-world arbitrage opportunities are "shades of grey" when compared to "perfect" arbitrage. Therefore, it is safe to say, real-world arbitrage opportunities are really subject to the judgements of the investor. But, one can better judge the viability of a less than perfect arbitrage opportunity by recalling the elements of perfect arbitrage. By doing so, we can better focus on the three primary aspects of any existing real-world, so called, arbitrage opportunity.

IV. Real-World Arbitrage Opportunities

If we accept the premise there is no such thing as a perfect arbitrage opportunity, we can turn our attention to real-world arbitrage opportunities. When an investor believes he or she has found a real-world arbitrage opportunity, he or she should exercise extreme caution and/or skepticism. The truth is, such true opportunities are few and far between. Often what looks like a real-world arbitrage opportunity is really not one because of such things as transactions costs, taxes, legal and/or institutional barriers, asset "lumpiness," maturity differences, lack of market size, and/or asset size differences that would erode any advantage(s). Furthermore, if such an opportunity does actually exist, it will most likely be exploited very quickly and, from a practical prospective, be gone before most investors can act. Large sophisticated financial organizations have computers, with real-time data inputs, that allow them to act quickly and appropriately; much more so than most individuals. Therefore, these types of organizations are normally the entities that make arbitrage profits.

V. How to Make an Arbitrage Profit

If an investor sees what he or she believes is an arbitrage opportunity, the question becomes, how does he or she make an arbitrage profit? First, if there is such an opportunity, the investor must be confident that at least one (and maybe more than one) of the associated assets is mispriced. As discussed previously, asset mispricing comes about because of market imperfections. Therefore, it would be reassuring if the investor knew the market imperfection or imperfections causing the asset mispricing, but this is not necessary for making arbitrage profit. The investor does not even need to know which asset or assets is or are mispriced. The only really important thing for the investor to know is which side of the deal is the cheap side and which side is the expensive side. Often this is easier said than done. But, it usually comes down to the cheap side giving the investor "more bang for the buck" than the expensive side. For example, if there are two loaves of bread for sale at the same price and all aspects of the two loafs are equal except one has 10 slices of bread and the other has 12 same size, etc. slices, which loaf is the cheaper? The 12-slice loaf is cheaper because the buyer gets two more slices of bread. Taking this example one step further, one can visualize a buyer buying the 12-slice loaf and selling the two extra slices to end up with a 10-slice loaf at a lower net price than what the store is charging for the equivalent 10-slice loaf. In this case, the arbitrage profit would be the difference between the store price for the 10-slice loaf and the net price paid by the buyer.

Assuming the investor can identify the cheap side and the expensive side of the deal, he or she will want to "sell" the expensive side and "buy" the cheap side. Theoretically, the order or these transactions is important because, ideally, the investor will have none of his or her money in the deal. Proceeds from selling the expensive side are used to buy the cheap side. In English, often the words used in place of sell are: short, borrow, write, float and words used in place of buy are: invest, long, lend, etc.

The possible arbitrage profit is typically the difference between the expensive side and the cheap side, depending on the point in time when the arbitrage profit is calculated. On a final note, if the associated calculations produce a negative arbitrage profit, it is usually because the cheap side was sold in order to buy the expensive side.

VI. Example Scenarios

A) Exchange Rate Arbitrage

Assume the following three exchange rates exist simultaneously and a potential arbitrager can move the various currencies without transaction costs, taxes, etc.

1 £ = 1.5 $ 1 € = 1.2 $ and 1 £ = 1.3 €

If the investor were to exchange $1,000 for 666.67 £ (via first exchange rate), then exchange 666.67 £ for 866.67 € (via the second exchange rate), and finally exchange 866.67 € for $1040 (via the third exchange rate), he or she would realize a $40 arbitrage profit.

<u>Some observations</u>:

1) If the investor were to exchange $1,000 for 833.33 € (via second exchange rate), then exchange 833.33 € for 641.13 £ (via the third exchange rate), and finally exchange 641.13 £ for $961.54 (via the first exchange rate), he or she would have a loss of $38.46. This would be an example of selling the cheap side and buying the expensive side as stated previously.
2) If the investor has no money, he or she could still exploit the arbitrage opportunity by initiating three futures or forward contracts matched with respect to size, execution, etc. This assumes there is no cost associated with entering into features or forward contracts. As the exchange rates would change over time, mark-to-market requirements would provide the investor with the arbitrage profit.
3) As the arbitrage activity described above continues, the exchange rates will converge until there is equilibrium (i.e. arbitrage opportunities are no longer available).
For example, the original exchange rates may become: 1 £ = 1.45 $, 1 € = 1.16 $, and 1 £ = 1.25 €. These exchange rates would provide no arbitrage opportunity.
Exchange $1,000 for 689.655 £ (via first exchange rate), then exchange 689.655 £ for 862.069 € (via the second exchange rate), and finally exchange 862.069 € for $1,000 (via the third exchange rate), he or she would realize zero arbitrage profit.

B) Yield Curve Arbitrage

1. If the current spot yield on a six-year discount note is 5 percent and the current spot yield on an eight-year discount note of similar default risk is 6 percent, what is the implied two-year forward rate for this risk category that begins in six years?

   $$\mathbf{(1 + .06)^8 = (1 + .05)^6 * (1 + i)^2 \rightarrow 1.594/1.340 = 1.1895522 = (1 + i)^2 \rightarrow \underline{\mathit{i = 9.04\%}}}$$

   This answer would make a potential investor indifferent with respect to investing for eight years or investing for six years plus rolling the six-year proceeds into a two-year instrument yielding 9.04 percent because the resulting cash at eight years would be the same.
2. If a bank believes the answer to question 1 is unrealistically high, how would the bank take advantage of the current pricing to make an arbitrage profit? Assume each note has a $1000 face value and please indicate the amount of profit, if the spot yield for two-year notes, six years from now, is 7 percent.

   a) Sell (short, borrow, write, float): $1000 @ 5% for 6 years
   b) Buy - Invest (long, lend,): $1000 @ 6% for 8 years
   c) Lock-in with a forward contract: borrow $1340, 6 years from now @ 7% for 2 years
   d) At the six year point, execute the forward contract (step iii) and collect $1340
   e) At the six year point, pay-off the first loan (step i) with $1340 collected from step iv
   f) At the eight year point, collect $1594 from step ii
   g) At the eight year point, pay-off the second loan (step iv) with $1534.17 of the $1594
   h) This yields an arbitrage profit of $1594 - $1534.17 or <u>**$59.83**</u> in the eighth year

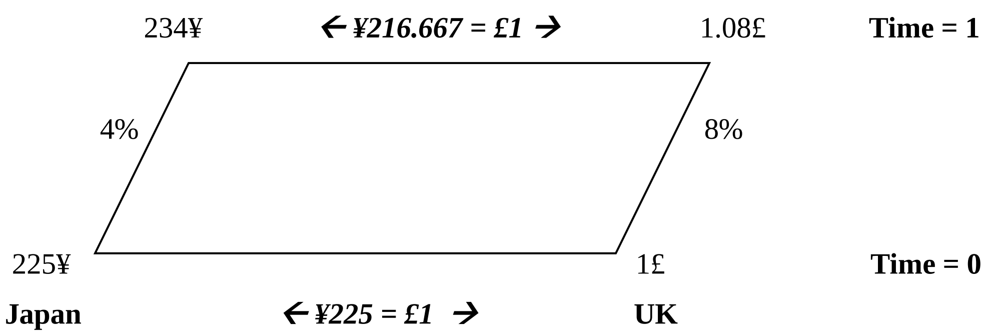
3. Is your answer to question 2 a perfect arbitrage? Please explain.

   No, there is still risk, unless you can do step c). Without that, the spot two-year rate, six years from now may change. If it goes above 9.04%, the bank will suffer a loss. With this risk, the bank is "speculating" on future interest rates. In addition, if there is default risk in any of the transactions, this adds risk to the situation.

C) Interest Rate Parity Theorem

1. Consider the following data in conjunction with the interest rate parity theorem: The current exchange rate between the Japanese yen (¥) and the UK pound (£) E_o is ¥225 to £1. The risk free rate in Japan is 4 percent while the risk free rate in the UK is 8 percent. Please calculate the implied one-year forward exchange rate for contracts executed one year from now. [Hint: Please express the one-year forward exchange rate in terms of yen to the pound (¥/£).]

£1(1 + .08) = £1.08 - one year from now. ¥225*(1 +.04) = ¥234 – one year from now*
Therefore, for investors to be indifferent, they must be equivalent at that time. For that to be true, the forward exchange rate must be: ¥234/£1.08 or **£1 = ¥216.667**

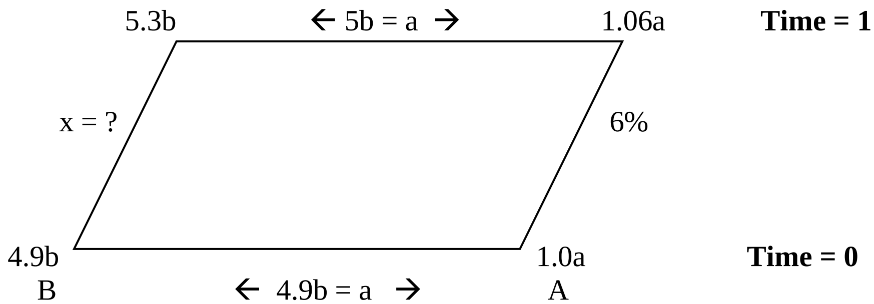


2. Suppose, that in question 1, the one-year forward exchange rate is ¥210 to 1£. How would an arbitrageur take advantage of this lack of equilibrium in the assumed interest rate parity theorem? [Hint: First (a) tell the general approach used by arbitrageurs and then (b) tell the potential profit, in pounds (£) on such a transaction originally valued at £5 million.] Please show your work.

Always, sell the expensive side and buy the cheap side. Therefore, in this case, ***1)*** *borrow £5 million @ 8% for 1 year in the UK,* ***2)*** *convert to yen at the current exchange rate,* ***3)*** *invest ¥1125 million @ 4% for 1 year in Japan,* ***4)*** *lock-in with a forward contract to convert ¥1170 million into £5.57143 million,* ***5)*** *in 1 year, collect ¥1170 from step 3 investment,* ***6)*** *execute forward contract (step 4),* ***7)*** *pay-off UK loan with £5.4 million,* ***8)*** *keep the difference,* ***£171,429*** *as arbitrage profit at the end of the year.*

3. The interest rate parity theorem is sometimes referred to as the purchasing power parity theorem. Often, one is asked to solve for the equilibrium forward exchange rate as in the example above. However, a related problem could ask us to solve for one of the other three variables, if the forward exchange rate and two others are known.

For example, suppose the current risk free rate in country A is 6%, the current spot exchange rate between country A and B's currency is 4.9b = a and the one-year forward exchange rate is 5b = a. What must the risk free rate in country B be for equilibrium to exist?



One unit of currency a can yield 1.06a in one year. In one year, 1.06a can be exchanged at the one-year forward rate of a = 5b to yield 5.3b. Therefore, if 1a can be exchanged for 4.9b currently, what risk free rate (x) must prevail in country B for there to be no arbitrage opportunity?

$(1 + x)*4.9\ b = 5.3b$ *so "x" must equal 8.16%.*

If this is the case, investors are indifferent as to how or where they invest because they will end up with 5.3b at the end of one year.

D) Put/Call Parity Theorem

The Put/Call Parity Theorem takes the form of the following equation:

$$C + X/(1 + R) = S_o + P$$

Where: C = current price of a one-year European call option with a strike price equal to S_o,

X = strike price of the call option (S_o),
R = risk free rate for one year,
S_o= current price of one share of the stock in question,
P = current price of a one-year European put option with a strike price equal to S_o.[3]

1. If the current prices are: C = \$2.00, R = 4%, S_o = \$25.00, P = \$4.00 How much, on a per share basis, does an arbitrageur (investor) stand to gain?

\$2.00 + \$25.00*(1/1+.04) = \$25.00 + \$4.00
\$2.00 + \$24.04 = \$25.00 + \$4.00
\$26.04 ≠ \$29.00

Therefore, an arbitrageur can gain \$2.96 per share by short selling the stock, writing the put, buying the call and loaning \$24.04 at the risk free rate.

2. Please explain how an arbitrageur would "close the position" to preserve his or her gains in question 1.

The investor would either use the call plus the \$25 to replace the short stock sale and the put would expire unexercised (if the stock appreciated in price) OR let the call expire unexercised and honor the put by buying the stock for \$25.00. Then replace the short stock sale (if the stock depreciated in price).

[3] It should be noted, arbitrage opportunities may exist, even if the discount rate is not the risk-free rate and/or the put and call are not 'on the money' options, as long as the dollar amount borrowed or loaned is equal to the strike prices of the two options.

3. If the current prices are: C = \$5.00, R = 4%, S_o = \$35.00, P = \$3.00 How much on a per share basis does an arbitrageur (investor) stand to gain?

\$5.00 + \$35.00*(1/1+.04) = \$35.00 + \$3.00
\$5.00 + \$33.65 = \$35.00 + \$3.00
\$38.65 ≠ \$38.00

Therefore, an arbitrageur can gain \$0.65 per share by writing the call, borrowing \$33.65 at the risk free rate, buying the stock and buying the put.

4. Please explain how an arbitrageur would "close the position" to preserve his or her gains in question 3.

The investor would either use the put to sell the stock for \$35.00. Then repay the loan with the \$35.00 and let the call expire unexercised (if the stock depreciated in price) OR let the put expire unexercised and repay the loan with \$35.00 of the proceeds from selling the stock when the call is exercised (if the stock appreciated in price).